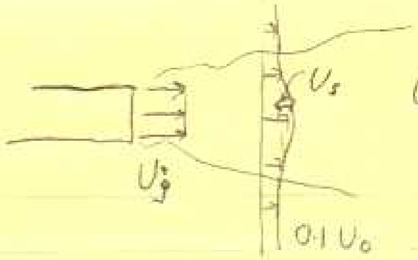
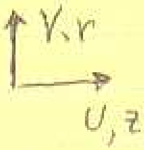
4.1 Axisymmetric Jet



$U = \hat{U}_s + 0.1\,U_0$

radial coordinates



$$U_\theta = 0, \quad \partial/\partial\theta = 0$$

streamwise momentum: $\rightarrow$ $U\dfrac{\partial U}{\partial z} + \overline{u\dfrac{\partial u}{\partial z}} + V\dfrac{\partial U}{\partial r} + \overline{v\dfrac{\partial u}{\partial r}} = 0$

continuity $\times u$ and average $\rightarrow$ $\overline{u\dfrac{\partial v}{\partial r}} + \dfrac{\overline{uv}}{r} + \overline{u\dfrac{\partial u}{\partial z}} = 0$

use continuity in mom. eq'n to get

$$U\frac{\partial U}{\partial z} + V\frac{\partial U}{\partial r} + \frac{\partial}{\partial z}\overline{uu} + \frac{1}{r}\frac{\partial}{\partial r}r\overline{uv} = 0$$

scale: $U\dfrac{\partial U}{\partial z} \sim \dfrac{U_s^2}{L} = \dfrac{U_s^2}{L}\cdot\dfrac{\ell}{u^2}\cdot\dfrac{u^2}{\ell}$

$V\dfrac{\partial U}{\partial r} \sim \dfrac{U_s \ell}{L}\dfrac{U_s}{\ell} = \dfrac{U_s^2}{L}\cdot\dfrac{\ell}{u^2}\cdot\dfrac{u^2}{\ell}$

$\dfrac{\partial\,\overline{uu}}{\partial z} \sim \dfrac{u^2}{L} = \dfrac{\ell}{L}\cdot\dfrac{u^2}{\ell}$

$\dfrac{1}{r}\dfrac{\partial}{\partial r}r\overline{uv} \sim \dfrac{u^2}{\ell}$

neglect 3rd term, retain others so long as $\left(\dfrac{U_s}{u}\right)^2\dfrac{\ell}{L} = O(1)$; $\dfrac{u}{U_s} \sim \left(\dfrac{\ell}{L}\right)^{1/2}$

rewrite: $\boxed{\dfrac{\partial UU}{\partial z} + \dfrac{1}{r}\dfrac{\partial}{\partial r}rUV + \dfrac{1}{r}\dfrac{\partial}{\partial r}(r\overline{uv}) = 0}$

momentum integral constraint: $U(2\pi r)dr$ is volume flux thru plane $\perp x$

where $U = 0.1U_0 + U_s$

ρU is mean momentum/unit volume

in momentum eq:

$$\frac{\partial}{\partial z}(0.1U_0 + \tilde{U}_s)^2 + \frac{1}{r}\frac{\partial}{\partial r}\left(rV(0.1U_0 + \hat{U})\right) + \frac{1}{r}\frac{\partial}{\partial r}(r\overline{uv}) = 0$$

integrate over area

$$\frac{\partial}{\partial z}\int_{-\infty}^{\infty}(0.1U_0 + \hat{U}_s)^2\, 2\pi r\, dr + 2\pi r V(0.1U_0 + \tilde{U}_s)\Big|_{-\infty}^{\infty} + 2\pi r\,\overline{uv}\Big|_{-\infty}^{\infty} = 0$$

$\lim_{r\to\pm\infty} \overline{uv} = 0 \qquad \lim_{r\to\pm\infty} U_s = 0$

what about rV? $\rightarrow$ continuity $\frac{1}{r}\frac{\partial}{\partial r}(rV) + \frac{1}{r}\frac{\partial}{\partial z}(rU) = 0$

integrate $2\pi r V \Big|_{-\infty}^{\infty} + 2\pi \frac{d}{dz}\int_{-\infty}^{\infty} r\,(0.1\,U_0 + \hat{U}_s)\,dr = 0$

so $\lim_{r\to\pm\infty} rV = -\frac{d}{dz}\int_{-\infty}^{\infty} r\,(0.1\,U_0 + \tilde{U}_s)\,dr$

put this back in momentum integral – other terms all $\rightarrow 0$

$$\frac{d}{dz}\int_{-\infty}^{\infty} r\,U_s\left(0.1\,U_0 + \hat{U}_s\right)dr = 0$$

$$\int_{-\infty}^{\infty} U_s\,(0.1\,U_0 + \tilde{U}_s)\,dr = \text{constant in } z$$

self-preservation $\rightarrow$ $\tilde{U} = U_s f(\xi)$, $-\overline{uv} = U_s^2 g(\xi)$ $\qquad \xi = \frac{r}{\ell}$

into momentum integral

$$\int_{-\infty}^{\infty} r\,U_s f\,(0.1\,U_0 + U_s f)\,dr = \text{constant}$$

$$0.1\,U_s U_0\,\ell^2 \int_{-\infty}^{\infty} \xi f\,d\xi + U_s^2 \ell^2 \int_{-\infty}^{\infty} \xi f^2\,d\xi = \text{constant}$$

to be independent of z

1) $0.1\,U_0 U_s\,\ell^2 = \text{const}$

2) $U_s^2 \ell^2 = \text{const}$

near $z = 0$, $U_s \gg 0.1\,U_0$, use 2)

$U_s^2 \ell^2 = \text{const}$

if $\ell \sim z^n$, $U \sim z^{-n}$

for jet, $z \gg 0$, $U_s \ll 0.1\,U_0$, 1) more important

$U_s \ell^2 = \text{const}$

if $\ell \sim z^m$, $U \sim z^{-2m}$

get n, m from momentum equation

$$0.1\,\frac{U_0\,l}{U_s^2}\frac{dU_s}{dz}f - 0.1\,\frac{U_0}{U_s}\frac{dl}{dz}f'\zeta + \frac{l}{U_s}\frac{dU_s}{dz}f^2$$

$$-\frac{dl}{dz}ff'\zeta + \frac{dl}{dz}f'\int_0^r f'\zeta\,d\zeta - \frac{l}{U_s}\frac{dU_s}{dz}f'\int_0^r f\,d\zeta$$

$$+ g' + g = 0$$

1. near $z \approx 0$, $U_s \gg .1U_0$, neglect 1st 2 terms and

require $\frac{dl}{dz}$ = constant ; $\frac{l}{U_s}\frac{dU_s}{dz}$ = constant

with $\cancel{z}$ $l \sim z^n$, $U_s \sim z^{-n}$ $\rightarrow$ $n = 1$

$$\boxed{l \sim z \quad ; \quad U_s \sim z^{-1}}$$

like a round jet with no ambient flow.

2. $z \gg 0$, $U_s \ll U_0$, consider only 1st 2 terms

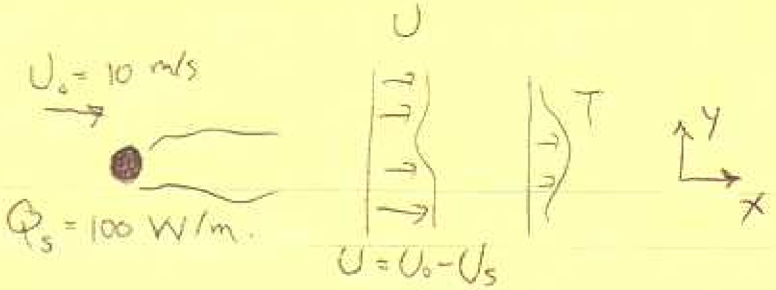
$\frac{l}{U_s^2}\frac{dU_s}{dz}$ = const, $\frac{1}{U_s}\frac{dl}{dz}$ = const

with $l \sim z^m$, $U_s \sim U^{-2m}$ $\rightarrow$ $m = 1/3$

$$\boxed{l \sim z^{1/3} \quad ; \quad U_s \sim z^{-2/3}}$$

like an axisymmetric wake

4.2. Heated Wake



With no sourses/sinks of heat in x-direction, heat is conserved if heat flux in plane $\perp x$ is constant at all x

i.e. $$\int_{-\infty}^{\infty} T(x,y)\, U(x,y)\, dy = \frac{Q_s}{\rho C_p}$$

self-similar wake $\rightarrow$ $U(\xi) = U_0 - U_s(x)\, e^{-\xi^2/2}$; $T(\xi) = T_s(x)\, e^{-\xi^2/2}$; $\xi = y/l$, $l = l(x)$

plane wake result $\rightarrow$ $U_s = 1.58\sqrt{\frac{\theta}{x}}\, U_o$; $l = .252\sqrt{\frac{x}{\theta}}\, \theta$

for cylinder $\rightarrow$ $\theta = \frac{d}{2} = 5\times10^{-4}$ m

then $$\int_{-\infty}^{\infty} T(x,y)\, U(x,y)\, dy = \int_{-\infty}^{\infty} U_o T_s e^{-\xi^2/2} dy - \int_{-\infty}^{\infty} U_s T_s e^{-\xi^2} dy = \frac{Q_s}{\rho C_p}$$

to integrate. let $\alpha = y/l$, $\beta = \sqrt{2}\, y/l$

result $\rightarrow$ $$T_s l \left(U_o - \frac{U_s}{\sqrt{2}}\right) = \frac{Q}{\sqrt{2\pi}\, \rho C_p}$$

replace l, U_s with plane wake x dependencies

Q? $\rightarrow$ at what x is $T_s^{rms} = 1°C$? since $U_{rms} \simeq .35\, U_s$; $T_s^{rms} \simeq .35\, T_s$

or $T_s = 1/.35\ °C \sim 3°C$

$$\boxed{x \simeq 0.05\text{ m} = 50\text{ mm} = 50\, d}$$