T&L 1.2

Box of L^3 volume filled with decaying turbulence.

large eddy scale $\ell \sim L$

$\epsilon \sim u^3/L$, $Re \gg 10$

$\therefore \frac{d}{dt}\left(\frac{3}{2}u^2\right) \simeq -\frac{u^3}{L}$, (rate of change of TKE $= \epsilon$)

$3\,u\,du \triangleq -\frac{u^3}{L}\,dt$

$$\int_{u(0)}^{u(t)} \frac{du}{u^2} \simeq -\frac{1}{3L}\int_0^t dt$$

$$\frac{1}{u(0)} - \frac{1}{u(t)} \simeq -\frac{t}{3L}$$

$$\frac{1}{u(t)} \simeq \frac{1}{u(0)} + \frac{t}{3L}$$

$$u(t) \simeq \frac{3L\,u(0)}{3L + t\,u(0)}$$

$$\therefore \boxed{\frac{3}{2}u^2(t) \simeq \frac{3}{2}\left[\frac{3L\,u(0)}{3L + t\,u(0)}\right]^2} \quad , \; R \gg 10$$

at $Re = 10$, $\epsilon = \frac{u^3}{L} = c\,\nu\frac{u^2}{L^2}$

$$c = \frac{u^3 L^2}{\nu u^2 L} = \frac{uL}{\nu} = Re = 10$$

$$\boxed{c = 10}$$

final period of decay, $Re < 10$, eddies decay directly by viscosity

and $\epsilon = 10\,\nu\,\frac{u^2}{L^2}$

$$\frac{d}{dt}\left(\frac{3}{2}u^2\right) = -10\,\nu\frac{u^2}{L^2}$$

$$\frac{3u\,du}{u^2} = -10\frac{\nu}{L^2}\,dt$$

$$\int_{u(0)}^{u(t)} \frac{du}{u} = -\frac{10}{3}\frac{\nu}{L^2}\,dt$$

$$\ln \frac{u(t)}{u(0)} = -\frac{10\nu t}{3L^2}$$

$$u(t) = u(0)\exp\left[-\frac{10\nu t}{3L^2}\right]$$

$$\tfrac{3}{2}u^2(t) = \tfrac{3}{2}u^2(0)\exp\left[-\frac{20\nu t}{3L^2}\right]$$

note: since $C = 10 = Re = \frac{u(0)L}{\nu}$ at $t=0$, $u(0) = 10\frac{\nu}{L}$

given $L = 1\,m$, $\nu = 15\times10^{-6}\ m^2/s$, $u = 1\ m/s$ at $t=0$

how long before $Re = 10$?

$$Re = \frac{u(t)L}{\nu} = \frac{u(0)L^2}{\nu(L+\frac{1}{3}u(0)t)}$$

$$Re\left(\nu(L+\tfrac{1}{3}u(0)t)\right) = u(0)L^2$$

$$\tfrac{1}{3}Re\,\nu\,u(0)\,t = u(0)L^2 - Re\,\nu L$$

$$t = \frac{3[u(0)L^2 - Re\,\nu L]}{Re\,\nu\,u(0)} = \frac{3L^2}{Re\,\nu} - \frac{3L}{u(0)}$$

$$= \frac{3(1)(1^2)}{10(15\times10^{-6})} - \frac{3(1)}{(1)} = (2\times10^4 - 3)\text{ seconds}$$

so $\boxed{t = 2\times10^4\,s \sim 5.6\text{ hrs.}}$

Effect of walls: can estimate effect by estimating thickness of molecular b.l. adjacent to wall.

define $Re = \frac{u_* \delta}{\nu}$, $u_* = \sqrt{\frac{\tau}{\rho}}$ is friction velocity

δ is b.l. thickness.

taking $Re \leq 10$ for laminar sublayer and $u_* = \frac{u}{30}$ (smooth walls) initially we have $10 = \frac{u\,\delta}{30\,\nu}$; $\delta = 300\,\frac{\nu}{u}$

or $\delta = \frac{300\,(15\times10^{-6})}{1} = 4.5$ mm.

So total volume <u>initially</u> occupied by laminar sublayer is negligible compared to volume of box

however, $\delta \propto \frac{1}{u}$ and we get an estimate for u at beginning of final decay period by $Re = \frac{uL}{\nu} = 10$

$$u = 10\,\frac{\nu}{L} = 1.5\times10^{-4} \text{ m/s}$$

so at period of final decay:

$$\delta = \frac{300\,(15\times10^{-6})}{15\times10^{-5}} = 30 \text{ m.}$$

So, long before $Re < 10$, δ approaches L and b.l. effects <u>will</u> become important. This may account for the rather long time (5.6 hrs) that we estimated for final decay period based upon model <u>without</u> b.l. effects.