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Resonant interaction of sound wave with internal solitons in the coastal zone

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Naturally occurring internal solitary wave trains (solitons) have often been observed in the coastal zone, but no reported measurements of such solitary waves include low-frequency long-range sound propagation data. In this paper, the possibility that internal waves are responsible for the anomalous frequency response of shallow-water sound propagation observed in the summer is investigated. The observed transmission loss is strongly time dependent, anisotropic, and sometimes exhibits an abnormally large attenuation over some frequency range. The parabolic equation (PE) model is used to numerically simulate the effect of internal wave packets on low-frequency sound propagation in shallow water when there is a strong thermocline. It is found that acoustic transmission loss is sensitive to the signal frequency and is a "resonancelike" function of the soliton wavelength and packet length. The strong interaction between acoustic waves and internal waves, together with the known characteristics of internal waves in the coastal zone, provides a plausible explanation for the observed anomalous sound propagation in the summer. By decomposing the acoustic field obtained from the PE code into normal modes, it is shown that the abnormally large transmission attenuation is caused by "acoustic mode-coupling" loss due to the interaction with the internal waves. It is also shown that the "resonancelike" behavior of transmission loss predicted by the PE analysis is consistent with mode coupling theory. As an inverse problem, low-frequency acoustic measurements could be a potential tool for remote-sensing of internal wave activity in the coastal zone.

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INTRODUCTION

Naturally occurring internal wave packets have often been observed in the coastal zone, especially in the summer. The mechanism for the generation of these nonlinear internal waves have been widely investigated in the geophysics and fluid mechanics community. Unfortunately, however, no reported measurements of such solitary waves include low-frequency long-range sound propagation data. The acoustic community has paid little attention to the effect of solitary waves on sound propagation with the exception of the work of Baxter and Orr that was based on ray theory and calculated the influence of an oceanic internal wave packet on short-range (high-frequency) sound propagation.\(^1\) Experiments, conducted by Zhou and his group at Institute of Acoustics of the Chinese Academy of Sciences in Beijing over a four-year period at the same area of Yellow Sea, have shown that the frequency response of shallow water sound propagation in the summer is a strong function of time and propagation direction, and sometimes exhibits an abnormally large attenuation over some frequency range. A part of these results was reported before,\(^2\) but it cannot be explained by a conventional range-independent model of sound propagation using reasonable bottom acoustic parameters and an average sound-speed profile.

In this paper, we investigate the possibility that the anomalous propagation results are due to the presence of internal waves. First, we briefly discuss the characteristics of internal wave packets in the coastal zone. We then review the aforementioned experimental results of Zhou et al. that exhibited the anomalous frequency response. In Sec. III, we hypothesize that this anomalous, anisotropic frequency response is caused by the influence of internal wave packets. We support the hypothesis with numerical simulation results obtained using the parabolic equation (PE) propagation model. In Sec. IV, we decompose the acoustic field obtained by using the PE model into the normal modes, and show that the abnormally large transmission loss which occurs over certain frequency ranges is due to "mode-coupling" loss induced by the internal wave packets. In Sec. V, we show that "resonancelike" behavior of the attenuation is consistent with mode coupling theory.

I. CHARACTERISTICS OF INTERNAL WAVE PACKETS IN THE COASTAL ZONE

Internal waves have been observed almost everywhere in the ocean.\(^4\) In the open ocean they are best described as a stochastic phenomenon with a broadband frequency wave number spectrum.\(^5\) However, analysis of extensive data on internal waves in the coastal zone,\(^6-22\) has shown that these waves exhibit the properties of solitons. The experimental data includes: current and temperature measurements; vertical profiles from CTD, XBT, and acoustic echo sounding...
devices; ship's radar and satellite (or space shuttle) images obtained at optical and radar frequencies. We are primarily interested in the effect of internal solitons on long-range sound propagation in the coastal zone, and not the mechanisms for the generation or propagation of internal wave packets (which has been widely investigated in the geophysics and fluid mechanics communities). We thus limit our discussion of internal waves to the following summary of the relevant characteristics of internal wave packets in the coastal zone.

1. Internal waves in shallow water are frequently observed in deterministic groups (wave packets) with well-defined wavelengths that are describable as solitary waves (or solitons). These waves are usually observed in summer when they are trapped in a strong and shallow seasonal thermocline.

2. Solitons in shallow water \((h/\lambda_i \ll 1)\) are described to first order in wave amplitude by the Korteweg–de Vries (KdV) equation: \(A_t + C_0A_x + \mu A A_x + \delta A_{xxx} = 0\). The solitary waveforms often have a "sech" profile. When an initial solitary wave propagates, it often evolves more rank-ordered solitons and exhibits clear nonlinear and dispersive features such as higher-than-linear group velocity, and a decrease in wavelength and amplitude toward the rear of the packet.

3. The number of wave packets are highly correlated with the strength of the local tides: The maximum number occurs during spring tides and the minimum number occurs during neap tides. Whether or not they occur is critically dependent on the structure of the background shear and stratification profiles.

4. Solitary wave packets propagate shoreward, and are often generated by a tidally driven flow over sills, continental shelf edges, or other major variation in underwater topography.

5. Because of the shallowness of the summer seasonal thermocline and the large amplitude of the coastal internal waves, strong surface expressions of solitons have been observed with a variety of remote sensors, including photographs and synthetic aperture radar (SAR) from satellites and space shuttles.

Naturally occurring large-amplitude internal solitons have been reported in many coastal zones of the world such as: the Massachusetts Bay,\(^6\)–\(^8\) the New York Bight,\(^9\)–\(^22\) Gulf of California,\(^10\),\(^13\) Andaman Sea offshore Thailand,\(^9\),\(^18\) the Australian North West Shelf,\(^19\) the Sulu Sea between the Philippines and Borneo,\(^15\),\(^16\) off the coast of Portugal,\(^12\),\(^18\) off Hainan Island in the South China Sea and off the Strait of Gibraltar in Alboran Sea,\(^23\) the Scotian Shelf off Nova Scotia,\(^14\) the Celtic Sea,\(^17\) and so on. They have also been observed in lakes.\(^24\),\(^25\) As an example, an excellent record of an internal wave packet is shown in Fig. 1. It was obtained by Orr using high-frequency acoustic scattering in the Massachusetts Bay.\(^7\),\(^8\) The period of the waves is Doppler shifted by the ship's speed of about 2.5 kn. The seasonal thermocline is displaced by 30 m (arrow 1) and the stratified point scatterers (zooplankton?) at 30 to 40 m are displaced more than 20 m (arrows 2 and 3). The heavy acoustic backscattering in the vicinity of arrow 4 and extending in an oscillatory pattern throughout the figure is possibly caused by turbulent mixing in the thermocline. The heavy scattering near 75 m is probably caused by euphausiid and mysid shrimp. Figure 2, taken from the work of Osborne and Burch,\(^9\) shows how rough and smooth bands on the water surface can be caused by internal solitons. Such surface expressions have been detected by ship, satellite, and space shuttle in many areas. For example, Fig. 3, taken from the work of Liu,\(^21\) is a line drawing of the internal wave packets in the New York Bight, observed from Seasat satellite, which clearly illustrates the anisotropic characteristics of internal waves in the coastal zone. There are numerous wave groups, all propagating shoreward. Each group consists of many solitons. The number of groups often depends on the local tide strength and the density profile of water column, i.e., it is a function of time.

The characteristics of internal wave packets in the coastal zone summarized here, will be helpful in explaining the anomalous experimental results, introduced in the next section.

II. FREQUENCY RESPONSE OF SHALLOW-WATER SOUND PROPAGATION IN THE SUMMER

Zhou and his group at Institute of Acoustics, Academia Sinica (IAAS) have measured the frequency response of shallow-water sound propagation under the condition of a
strong thermocline in the Yellow Sea off China. A great deal of data was collected over several years in the same area, (which had a characteristically flat seabottom with high-speed sediments). The data included measurements of sound propagation, long-range reverberation, sound field spatial coherence and utilized normal-mode spatial filtering techniques. The acoustic parameters of the bottom for this area were obtained by wideband acoustic measurements. Generally speaking, the experimental data fits theoretical predictions very well. For example, transmission losses as a function of frequency for winter, late spring, and sometimes for summer were correctly predicted. However, it was found that, in the summer, even along the same experimental track, with similar averaged sound-speed profiles the frequency response of sound transmission could often be very different for different years.

Figure 4 shows power spectra that are obtained by averaging several explosive propagation signals. The upper curve is the power spectrum at a propagation range of 0.5 km, the lower one is at a range of 28 km. The difference between the two curves is a measure of the sound transmission loss between two receiving points, i.e., the frequency response of sound propagation. Between 300 and 1100 Hz, especially around 600 Hz, the transmission loss is abnormally large. The measured sound-speed profiles at the receiving ship are shown in Fig. 5. Both source and receiver were located below the thermocline. The anomalous transmission loss cannot be explained by conventional models of sound propagation (with a full complement of bottom acoustic parameters and an averaged sound-speed profile of water column). In order to explain this phenomenon, Zhou and his colleagues at IAAS made observations, every August for 4 years at same area. The abnormal attenuation frequency range varied with time. In Fig. 4, it occurs at around 600 Hz, but for different years or different propagation directions, it has been observed around 500, 1200, or 1600 Hz. Sometimes, there is no apparent abnormal attenuation frequency range.

In another experiment, the frequency response at a fixed range for six different radial directions was obtained. The results are shown in Fig. 6. The transmission distance was kept constant at 28 km, by moving the source ship along one quadrant of a circle centered at the receiving ship. Each curve represents an averaged value obtained from several explosive signals. Transmission loss is obviously a strong function of propagation direction. For different propagation directions, at some frequencies, the sound intensity varied as much as 25 dB! However, in another experiment, at the same

FIG. 3. Line drawing indicating internal wave packets in the New York Bight, observed from the Seasat satellite.

FIG. 4. Explosive signal power spectrum for shallow-water sound propagation in the summer which exhibits an abnormally large attenuation around 600 Hz (experimental data). Both source and receiver are located below the thermocline. Computational results (see Sec. III B): (1) without packet -- --. (2) with three packets O O O.
III. NUMERICAL SIMULATION OF THE INFLUENCE OF INTERNAL WAVE PACKETS ON SOUND PROPAGATION

A. Oceanic model

The presence of internal waves makes the sound-speed profile of the water range dependent. Unfortunately, in Zhou's experiments there were no accompanying systematic measurements of the internal wave field; and only the sound-speed profile at the receiver was measured. During the time period over which the frequency response shown in Fig. 4 was obtained, it was found that the thermocline depth at receiving ship location did change with time. Figure 5 shows three sound-speed profiles that were obtained at different times.

During the experiment, Zhang measured the temperature fluctuation as a function of time at the receiving ship. A record of temperature fluctuations over a period of 42 min at a fixed depth (around the thermocline) is shown in Fig. 7. The temperature fluctuated between about 13°C and 27°C. The possible presence of several individual solitary waves is indicated by temperature peaks due to the depression of the thermocline. Figures 5 and 7 give at least some evidence that there was solitary wave activity in the area of acoustic experiments.

Due to the fact that we have no data concerning specific characteristics of internal waves at the experimental site, for simplicity, following Lee's three-layer model of internal wave, we assume that the internal wave packet can be expressed by a gated sine function as shown in Fig. 8(b). In this figure, if a packet begins at $r_0$, then, for $r_0 + L_p > r > r_0$,

$$Z_1 = 14.0 - 2.0 \sin \left( \frac{2\pi r}{\lambda_1} \right),$$

(1)

$$Z_2 = 19.0 - 5.0 \sin \left( \frac{2\pi r}{\lambda_1} \right).$$

(2)

The idealized sound-speed profile in the absence of internal waves for the numerical simulation is shown in Fig. 8(a). We call $\lambda_1$ the soliton wavelength, and $L_p$ the packet (high-frequency) sound propagation. The calculations showed that at a fixed position the intensity of sound field could vary by as much as 20 dB, relative to a case without the presence of the internal wave packet. We hypothesized that internal wave packets might have a strong influence on low-frequency long-range sound propagation in shallow water as well and could possibly explain Zhou's Yellow Sea propagation data. In the next section, we support this hypothesis with numerical simulation results obtained using the parabolic equation method.
length. Referring back to Fig. 1, we see that an almost sinusoidal interface exists between the warmer surface layer and the underlying cooler water. The waveform of the soliton in Fig. 5 of Ref. 1 is also almost sinusoidal. The simplified, sinusoidal model, is thus seen to be not physically unreasonable. The main purpose of this work is to provide the first physical insight into the possible effects of internal wave packets on wideband low-frequency sound propagation in shallow water. It is not to predict the characteristics of internal waves for a specific area. Our simplification should not alter the qualitative results which follow.

The averaged values for bottom sound speed \( c_b \) and bottom acoustic attenuation \( \alpha_p \) for the experimental area are given by \(^{26,30}\)

\[
\frac{c_b}{c_w} = 1.056, \tag{3}
\]

\[
\alpha_p = 0.34f^{1.84} \text{ dB/m,} \tag{4}
\]

where \( f \) is the frequency in kHz.

**B. Comparison of numerical frequency response with experimental data**

Numerical simulation results obtained using the parabolic equation (PE) method (IFD\(^{31-33}\) code), show that internal wave packets can significantly change low-frequency transmission loss. The frequency response and, in particular, the abnormal attenuation frequency range are sensitive functions of the parameters of the soliton wave packets.

For example, using the simplified ocean model given in Fig. 8 and Eqs. (1)–(4), we put three internal wave packets located at 5, 15, and 25 km, along propagation track with each packet consisting of six solitons each with a wavelength \( \lambda_i \) of 235 m \( (L_p = 6\lambda_i = 1410 \text{ m}) \). The transmission loss as a function of range for four frequencies (with a receiver depth of 32 m and a source depth of 25 m) is shown in Fig. 9. The difference between the results with and without the soliton packets present is small at 100 or 1000 Hz, but at 600 Hz the difference at 30 km is as much as 25 dB. The most interesting and encouraging result is that, in this case the numerical transmission loss as a function of frequency (shown in Fig. 4 by the circles) fits the experimental results quite well.

As a consequence of the characteristics of internal wave packets described in Sec. 1 (propagation in groups, well-defined wavelengths, high correlation with tides, shoreward propagation), the projection of the solitons in different acoustic propagation directions and for different times will be different. If we change the soliton wavelength in the three packets to 235, 300, 350, 400, and 500 m, we would get frequency responses as shown in Fig. 10, that are similar to the experimental results for different propagation directions shown in Fig. 6. Hence, interaction between acoustic waves and internal wave packets is consistent with Zhou's experimental results for shallow-water sound propagation in the summer.

Why does the abnormally large attenuation in Fig. 4 occur around 630 Hz for internal wave packets consisting of six solitons with a wavelength of 235 m? Why do internal wave packets with the same parameters have a much smaller effect on sound propagation at frequencies around 300 or 1000 Hz? In the next section, we will show that these are due to "resonances" in the acoustic mode-coupling induced by the internal wave packets.

**IV. RESONANCE EFFECTS**

**A. Frequency resonance**

For our numerical model of shallow water, with a water depth of 38 m, mode stripping caused by seabottom attenuation, results in only a few modes still being present at a distance of 15 km. If the source and receiver are located below the thermocline, for a realistic sea bottom, the first mode dominates the sound field at long-range over the frequency range of interest. In order to isolate the effect of internal wave packets on the mode coupling among acoustic normal modes, we put a single packet consisting of six solitons with a wavelength of 235 m at 15 km and use the first normal mode alone as the initial input field to the parabolic equation code. Thus, prior to interaction with the packet, we assume only one mode (the first) is present; higher-order modes are generated as the first mode propagates through the internal wave packet. The depth distribution function of...
sound pressure amplitude at 15, 20, and 30 km obtained by PE method are shown in Fig. 11. At a range of 15 km (before the wave has interacted with the internal wave packet), the shape of the first mode is maintained at all frequencies. After interaction with the internal wave packet, at 300 and 1000 Hz, the energy is still predominantly in the first mode. However, for 630 Hz, the depth distribution shape is quite different from the first mode, due to resonance scattering, and the amplitude is much smaller.

The mode coupling due to interaction with internal wave packets can be easily calculated. Neglecting the evanescent modes, which decay at large range, leaves only a
finite number of propagating modes at a given frequency. The propagated field \( \Phi_{PE}(r,z) \) obtained by the PE method can be expanded in terms of a set of local modal eigenfunctions \( [U_n(z)] \), corresponding to the sound velocity profile shown in Fig. 8(a), as follows:

\[
\Phi_{PE}(r,z) = \sum_n A_{PE}(r) U_n(z),
\]

where the local modal eigenfunctions \( U_n(z) \) satisfy the appropriate boundary conditions at the sea surface and bottom and the differential equation

\[
\frac{d^2 U_n}{dz^2} + [k_n(z) - k_0^2] U_n = 0,
\]

where \( k_n \) is the eigenvalue (modal wave number). The modal eigenfunctions also satisfy the orthogonality relation

\[
\int \rho(z) U_n(z) U_m(z) dz = \delta_{n,m}.
\]

The Kronecker delta on the right side is unity if \( n = m \) and is zero if \( n \neq m \).

Multiplying both sides of Eq. (5) by \( U_m(z) \) and integrating over depth with help of Eq. (7) (mode spatial filtering), we obtain

FIG. 10. The computational transmission loss at 28 km for three internal wave packets with different wavelengths \( \lambda_i = (1)235 \text{ m}, (2)300 \text{ m}, (3)350 \text{ m}, (4)400 \text{ m}, (5)500 \text{ m} \).

FIG. 11. The depth distribution function of sound-pressure amplitude for different frequencies at a range of 15 km (- - -), 20 km (--), and 30 km (---) obtained using the PE method. PE input field: mode 1. One internal wave packet at 15-16.41 km with \( \lambda_i = 235 \text{ m} \).

(a) 300 Hz

(b) 630 Hz

(c) 1000 Hz
FIG. 12. The relative amplitudes of different modes at 18 km, showing frequency resonance of mode coupling. PE input field: mode 1. One packet at 15–16.41 km. \( \lambda = 235 \text{ m} \).

\[ A_{PEa}(r) = \int \rho(z) \Phi_{PE}(r,z) U_n(z) \, dz. \] (8)

After the interaction of the first mode with the internal wave packet, at selected ranges where no internal wave exists, the relative amplitude of higher-order mode to the first mode, \( A_{n1} \), is given by

\[ A_{n1} = \left( \frac{|A_{PEa}(r)|}{|A_{PE1}(r)|} \right) = \left( \frac{|\int \rho(z) \Phi_{PE}(r,z) U_n(z) \, dz|}{|\int \rho(z) \Phi_{PE}(r,z) U_1(z) \, dz|} \right)^{1/2} = \left( \frac{A_{nR} + A_{nI}}{A_{1R} + A_{1I}} \right)^{1/2}, \] (9)

with

\[ A_{nR} = \left( \sum_{j=1}^{m} \rho_1 \Phi_R(r,z_j) U_n(z_j) + \sum_{j=m+1}^{M} \rho_2 \Phi_R(r,z_j) U_n(z_j) \right)^2, \] (10)

\[ A_{nI} = \left( \sum_{j=1}^{m} \rho_1 \Phi_I(r,z_j) U_n(z_j) + \sum_{j=m+1}^{M} \rho_2 \Phi_I(r,z_j) U_n(z_j) \right)^2, \] (11)

where \( \Phi_R(r,z_j) \) and \( \Phi_I(r,z_j) \) are the real part and the imaginary part of PE field in the \( j \)th depth increment, \( M \) is the number of layers and the water-bottom interface occurs between the layer \( m \) and layer \( (m + 1) \). The quantity, \( A_{n1} \), is a measure of the energy exchanged between the first mode and higher-order modes. In this manner using a normal-mode computer program, a PE computer program and the ocean model described in Sec. III, we were able to decompose the sound field obtained from PE into normal modes. The input field for the PE code is the first normal mode. In the absence of modal coupling only this mode would ever be present. Results for three different frequencies are shown in Fig. 12. The plots show the relative modal amplitudes at 18 km. For ranges less than 15 km, only the first mode is present. As indicated in Fig. 12, beyond 15 km there is interaction with the internal wave packet, and at 630 Hz a significant amount of energy has been transferred from mode 1 into higher-order modes that will attenuate rapidly thereafter. Relatively little modal coupling occurs at the other two frequencies. Table I shows the eigenvalue \( k_n \) and the attenuation rate \( \beta_n \) (corrected for cylindrical spreading) for the first seven normal modes, calculated by a normal mode program for the sound velocity profile shown in Fig. 8(a) and a frequency of 630 Hz.
From this analysis, it is evident that the modal coupling caused by internal waves can sometimes be an important loss mechanism for sound transmission in shallow water in the summer. At 300 and 1000 Hz, the mode-coupling effect is much weaker. Only a few percent of the wave energy is coupled into high-order modes. At 630 Hz, however, a significant amount of energy has been transferred from mode 1 into higher-order modes, especially mode 3 and mode 4, with much larger attenuation rate. The mode-coupling induced by internal wave packets exhibits a frequency resonance effect, and can cause abnormally large transmission loss around the resonance frequency. The abnormally high attenuation observed by Zhou et al. over certain frequency ranges is consistent with interaction with internal waves. In Sec. V, we will discuss what determines the "resonance frequency."

### TABLE I. The eigenvalue $k_n$, attenuation rate $\beta_n$, and eigenvalue difference between modes 1 and $n$ for the first seven normal modes, $f = 630$ Hz.

<table>
<thead>
<tr>
<th>Mode $n$</th>
<th>$k_n$</th>
<th>$\beta_n$ (dB/km)</th>
<th>$k_1 - k_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.635 594 3</td>
<td>0.2454</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2.626 325 6</td>
<td>0.9169</td>
<td>0.009 268 7</td>
</tr>
<tr>
<td>3</td>
<td>2.611 159 6</td>
<td>1.7661</td>
<td>0.024 434 7</td>
</tr>
<tr>
<td>4</td>
<td>2.591 298 6</td>
<td>2.4771</td>
<td>0.053 845 3</td>
</tr>
<tr>
<td>5</td>
<td>2.572 735 4</td>
<td>2.8585</td>
<td>0.072 387 1</td>
</tr>
<tr>
<td>6</td>
<td>2.548 399 6</td>
<td>2.6948</td>
<td>0.087 194 7</td>
</tr>
<tr>
<td>7</td>
<td>2.534 173 7</td>
<td>2.3948</td>
<td>0.091 194 7</td>
</tr>
</tbody>
</table>

#### B. Soliton wavelength resonance

We continue to consider a single packet located at a distance of 15 km. We now fix the acoustic frequency at 630 Hz and the packet length ($L_p$) at 1.4–1.5 km and calculate the effect of soliton wavelength ($\lambda_s$) on transmission loss. Figure 13(a) shows the transmission loss as a function of soliton wavelength at a range of 30 km (with a receiver depth $r = 30$ km). Figure 13(b)–(d) shows the depth distribution of sound-pressure amplitude for different soliton wavelengths at 15 km, 20 km, and 30 km obtained by PE method. PE input field: mode 1. One packet at 15–16.41 km. $f = 630$ Hz.

![Fig. 13](image-url)

**FIG. 13.** (a) Soliton wavelength resonance. (b)–(d) The depth distribution function of sound-pressure amplitude for different soliton wavelengths at 15 km, 20 km, and 30 km obtained by PE method. PE input field: mode 1. One packet at 15–16.41 km. $f = 630$ Hz.
of 32 m and a source depth of 25 m). Transmission loss exhibits a resonancelike maximum at a soliton wavelength of 235 m. For other soliton wavelengths between 120 and 350 m, the difference between transmission loss with and without the presence of an internal wave packet is not significant. Taking first normal mode as the input field to the PE code, we obtained the depth distribution functions shown in Fig. 13(b)–(d). After interaction with the packet, for a soliton wavelength of 235 m, the shape is very different than that of the first mode, but for 200 and 280 m, the shapes are very close to that of mode 1.

At a distance of 18 km, we again decompose the PE calculated sound field into normal modes and get the results for three different soliton wavelengths shown in Fig. 14. For a soliton wavelength of 235 m, a significant amount of energy has been transferred from mode 1 into higher-order, higher-loss modes. However, for soliton wavelengths of 200 and 280 m, the mode-coupling effect is rather weak. That is, the mode coupling and, hence, the loss induced by internal wave packets exhibits a soliton wavelength resonance effect.

### C. Packet length resonance

Once again, considering a single packet located at a range of 15 km, we now hold acoustic frequency (630 Hz) and soliton wavelength (235 m) constant, and calculate the effect of internal wave packet length (or, equivalently, the number of solitons) on transmission loss. Figure 15(a) shows the transmission loss as a function of packet length at 30 km (with a receiver depth of 32 m and a source depth of 25 m). The transmission loss is a periodic, “resonancelike” function of the packet length ($L_p$). Taking first normal mode as the input field to the PE code, we obtain the depth distribution functions for the sound field for three packet lengths shown in Fig. 15(b)–(d). At 18 km, after interaction with the packet, the shape of the depth distribution function for a six-soliton packet is very different from that of the first mode. However, for a longer packet consisting of 11 solitons it is close to the shape of the first mode and close even in amplitude to the results without a packet.

At a distance of 18 km, we again decompose the PE sound field into normal modes, and get the results for two different packet lengths shown in Fig. 16. For a 1410-m packet (six 235-m solitons), a significant amount of energy has been transferred from mode 1 into higher-order modes. For a longer packet (2585 m, i.e., for 11 solitons), the mode-coupling effect is much weaker. That is, the mode-coupling and hence the loss induced by internal wave packets exhibits a packet length resonance effect.

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**FIG. 14.** The relative amplitudes of different modes at 18 km, showing soliton wavelength resonance of mode coupling. PE input field: mode 1. One packet at 15–16.41 km. $f = 630$ Hz.
V. MODAL COUPLING ANALYSIS

Dozier and Tappert,\textsuperscript{34} and McDaniel and McCom-\textsuperscript{35} have shown that the exchange of energy between acoustic modes $n$ and $m$ induced by random internal waves in the deep sea or lateral seabed inhomogeneities in shallow water exhibits a wave number resonance effect. The principal transfer of energy will occur between modes whose eigenvalue difference equals the wave number of the spectrum peak of the internal waves or sub-bottom roughness. This result is also applicable to our numerical results shown in Figures 11–14. Significant energy transfer will occur between modes $m$ and $n$ if

\begin{equation}
    k_{\text{int}} \approx k_n - k_m,
\end{equation}

where $k_{\text{int}} = 2\pi/\lambda_i$ is the wave number of the internal wave. [If the mode coupling is considered to be a quantum mechanical phonon-soliton interaction, Eq. (12) is a statement of conservation of momentum. The corresponding statement of conservation of energy $\omega_{\text{int}} = \omega_n - \omega_m$ can be shown to correspond, macroscopically, to the Doppler shift which results from scattering from the moving internal wave.] For our numerical model of internal wave packets with a wavelength of 235 m, the wavenumber of the spectrum peak of the internal wave $k_{\text{int}} = 2\pi/235 = 0.026737$. Comparing this value of $k_{\text{int}}$ with the eigenvalue differences between mode 1 and mode $n$ shown in Table I, it is found that $k_{\text{int}}$ is almost equal to the eigenvalue difference between acoustic normal...
modes 1 and 3. Thus one would expect a significant amount of energy to be transferred from mode 1 to mode 3 (but not, say, to mode 2). Moreover, \( \kappa_m \) is also reasonably close to the differences between the eigenvalues of modes 3 and 4 and between modes 4 and 6, etc. Hence, one would expect some transfer of energy from mode 1 to mode 4 via mode 3, or from mode 1 to mode 6 via modes 3 and 4, and so on. These simple arguments explain why a significant amount of energy is transferred from mode 1 into higher-order modes, especially modes 3 and 4, and why and where the acoustic wave frequency and soliton wavelength resonances occur.

The packet length resonance effect mentioned in Sec. IV is not so easily explained, and it is still not clear how to predict where it will occur. We present here a preliminary explanation for the phenomenon given by S. T. McDaniel.\textsuperscript{36} If the backscattered field is neglected, the equation governing mode coupling between \( m \)th and \( n \)th modes due to range dependence of the ocean environment, can be expressed as [see Eq. (14) in Ref. 37 for notation]

\[
\frac{du_m}{dr} = \frac{1}{2} \sum M_{nm} u_m \exp\left[i(\bar{\kappa}_m - \bar{\kappa}_n) r\right]. \tag{13}
\]

Let a resonant coupling condition be

\[
M_{nn} = -2K_L \exp\left[-i(\bar{\kappa}_m - \bar{\kappa}_n) \right] \tag{14}
\]

and \( M_{mn} = -M_{nm} \). We consider only two modes, modes 1 and 3, which are assumed to satisfy the resonant coupling condition. From Eqs. (13) and (14), we obtain

\[
\frac{du_1}{dr} = -K_L u_m, \tag{15}
\]

\[
\frac{du_m}{dr} = K_L u_1, \tag{16}
\]

or

\[
\frac{d^2u_{1,m}}{dr^2} + K_L^2 u_{1,m} = 0. \tag{17}
\]

If all of the energy is assumed to be initially in the first mode \( (u_1 = 1 \) and \( u_m = 0 \) ) the solution to Eq. (17) is

\[
u_1 = \cos(K_L r), \tag{18}
\]

\[
u_m = \sin(K_L r). \tag{19}
\]

Equation (19) predicts that the magnitude of the excited higher-order modes will be periodic with range. The resonant coupling parameter \( K_L \) determines for what packet lengths \( u_m \) has a maximum value (or \( u_1 \) has a minimum value). From Eq. (19), the packet length "resonance" will occur when \( K_L L = (n + 1/2) \pi \). From the first packet length resonance in Fig. 15(a), we have \( K_L L_p = K_L \times 235 \times 6 = \pi/2 \) and \( K_L = 1.11/\text{km} \); from the second resonance length, we have \( K_L L_p = K_L \times 235 \times 17 = 3\pi/2 \) and \( K_L = 1.18/\text{km} \). The two values obtained for \( K_L \) are almost the same. The agreement is quite satisfactory considering that the mode coupling occurred between more than just two modes (modes 1 and 3) and \( K_L \) would be changed with the coupling strength. The mode coupling is a periodic (resonance) function of the internal wave packet length, and, hence, acoustic transmission loss at long-range must vary with the packet length. It is shown that the "resonancelike" behavior of transmission loss predicted by the PE analysis in this paper is consistent with mode coupling theory.

**VI. CONCLUSION**

In summary, (1) we have briefly reviewed the characteristics of naturally occurring internal wave packets in the coastal zone which differ in many respects from open sea internal waves. We demonstrate that such internal wave packets can have a strong influence on low-frequency long-range sound propagation in shallow water. (2) The measured frequency response of sound propagation during the summer is often a strong function of both time and propagation direction, sometimes the sound propagation has an abnormally large attenuation over some frequency range. (3) Numerical calculations have shown that the interaction between the acoustic waves and internal wave packets and particularly resonance effects in the acoustic mode-coupling induced by internal wave packets could be an important loss mechanism for shallow-water sound propagation in the summer. Modal coupling induced by internal waves could explain the anomalous acoustic experimental results. (4) The fact that the acoustic mode-coupling induced by internal wave packets exhibits frequency, soliton wavelength, and packet length resonances [and perhaps existence of the Doppler shift alluded to in Sec. V, as well] suggest that low-frequency acoustic measurements could be used for remote monitoring of internal wave activity and extracting of hydrological and meteorological characteristics of the water mass in the coastal zone.

The analysis and numerical calculations presented here are based on a simplified oceanic model. Although the simplification should not alter the qualitative results presented, a more refined model would be desirable for a more detailed comparison with observations. It is, of course, also possible that some seabottom structure (for example, a surficial layer of low speed), mode-coupling due to the seabed roughness or sediment inhomogeneities and fish shoals could also produce an extra acoustic attenuation over some frequency range. It would thus be desirable for the acoustic, marine geology and geophysics communities including remote-sensing groups to work together to conduct joint at-sea experiments at a specific sea area. If data on internal wave activity (solitary or random), seabottom parameters, and sound propagation were obtained simultaneously and systematically, our understanding of the interaction between internal waves and sound waves in the coastal zone and low frequency acoustic propagation loss in shallow water would be greatly enhanced.

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